**Weak Precedence Grammars**

*Def:* A context-free grammar is a **weak precedence grammar** if

1. all right hand sides are unique,
2. there are no empty right hand sides,
3. the relation > is disjoint from the union of < and =, and
4. if \( A \rightarrow \alpha X \beta \) and \( B \rightarrow \beta \) are in \( P \) with \( X \in V \), then neither of the relations \( X < B \) nor \( X = B \) are valid.

The first two conditions are easily verifiable by simply inspecting the grammar. The third and fourth conditions require a little work. First, we will compute the simple precedence relations. Then we can easily verify that conditions 3 and 4 hold. Condition 3 is verified by observation of the simple precedence matrix. Condition 4 requires that we look at each right hand side for common tail symbols or common tail symbol strings. We then need to verify that condition 4 holds. This is tedious but doable. Fortunately, we have a program, *analyzer*, that will compute those weak precedence relations and verify conditions 3 and 4.

Consider the following grammar:

1. \( E \rightarrow E + T \)
2. \( | T \)
3. \( T \rightarrow T * P \)
4. \( | P \)
5. \( P \rightarrow (E) \)
6. \( | V \)

We see that the grammar has unique right hand sides and that there are no empty right hand sides. Hence, conditions 1 and 2 in the above definition are satisfied. We now turn our attention to conditions 3 and 4. For condition 3 we will compute the simple precedence relations. We will use the same computation technique that we used previously for our simple precedence relations. Thus, we get:

```
  E  T  P  +  *  (  )  V
  | | | | | | | |
E | | | | | | | |
  | | | | | | | |
T | | | | | | | |
  | | | | | | | |
P | | | | | | | |
  | | | | | | | |
+ | | LE | L | | | | |
  | | | | | | | |
* | | | | | | | |
  | LE | L | | | | |
( | LE | L | | | | | |
```

In looking at the matrix, we see that the only conflicts that we have are less than-equals conflicts. Hence, condition 3 is satisfied. We now turn our attention to condition 4. To see that condition 4 is not violated, we consider productions that have the same tail symbols, namely we consider productions 1 and 2 along with productions 3 and 4. Productions 1 and 2 both have T as the tail symbol while productions 3 and 4 have P as the tail symbols.

**Note:** We are only looking at a single symbol in our example. However, there could just as easily be a string of characters at the end of a right hand side that were in common in two or more productions. We need to consider those productions also.

**Note:** It is a coincidence that the productions that we are looking at have the same left hand side. Same left hand sides are not a requirement.

Let's look at productions 1 and 2. Repeating condition 4 from above

if \( A \rightarrow \alpha X \beta \) and \( B \rightarrow \beta \) are in \( P \) with \( X \in V \), then neither of the relations \( X < B \) and \( X = B \) are valid.

and productions 1 and 2, we have

1. \( E \rightarrow E + T \)
2. \( T \)

Hence, if we have a relation between + and E, then the weak precedence condition 4 is not satisfied. Looking at the matrix above we see that there is no relation between + and E. We next consider productions 3 and 4. We have

3. \( T \rightarrow T * P \)
4. \( P \)

In this instance if we have a relation between * and T, then we do not have a weak precedence grammar. Again looking at the matrix, we see that there is no relation between * and T. Hence, all four conditions are satisfied, and the grammar is a weak precedence grammar.