

Solutions to Practice Test 1

1. Consider K_3 .
2. Consider any person x . Let A be their acquaintances and S their strangers. At least one of these sets has three members; say A . If any two of A know each other, we are done (trio of x and these two of A). But if none of A knows each other, we are also done (A is desired trio).
3. Petersen graph has 10 vertices of odd degree. Therefore 5 is necessary (and easily shown to be sufficient; for example, we showed that can decompose into 5 P_4 's).
4. No each time (sum of degrees is odd).
5. Let v be a vertex in the center. If we consider any two other vertices x and y , there is a walk from x – y by taking first the shortest path from x to v and then the shortest path from v to y . Thus, distance from x to y is at most twice v 's eccentricity, which is the radius.
6. (a) 2^7 (b) 55 (Fibonacci numbers)
7. Construct a maximal path. Says ends at vertex v . This vertex has at least k neighbors, all on the cycle; so go to the farthest and then come back along the path, to obtain the desired cycle.
8. Edges are: 2-5, 6-1, 1-3, 7-5, 5-4, 4-3, 3-8
9. Edges are: 1-2, 2-3, 3-4, 4-5, 5-6, 6-7, 7-8. Total weight is 63.