Summary of West 6: Planar graphs

A graph is planar if it has a drawing without crossings (a planar embedding). Neither K_5 nor $K_{3,3}$ is planar. (Note that a full proof needs Jordan curve theorem.)

A plane graph is a planar graph already drawn in the plane; the drawing divides the plane into faces. The dual of a plane graph has one vertex for each face of the original, and two vertices are adjacent if their faces share an edge-boundary.

A graph is outerplanar if it has an embedding in the plane with all the vertices on the outer face. Neither K_4 nor $K_{2,3}$ is outerplanar. Minimum degree of outerplanar graph is at most 2.

Euler's formula says: $n-e+f = 2$ for connected plane graph. Thus: $e \leq 3n-6$ and $\delta \leq 5$. A maximal planar graph is one where every face is a triangle.

Kuratowski's theorem says that a graph is planar if and only if it does not contain a subdivision of K_5 nor $K_{3,3}$. (Proof omitted.)

Kempe showed that every planar graph has chromatic number at most 5. The 4-color theorem of Appel, Haken, Koch used computers and showed that every planar graph is 4-colorable.