MthSc 853 — Goddard — Fall09

Practice Test 2

1. Give the trace and determinant of the following matrix:

$$\begin{pmatrix} a+b & c & c \\ a & b+c & a \\ b & b & a+c \end{pmatrix}$$

2. Consider the complete undirected graph D_n: this is the graph with n vertices such that there is one edge between every pair of vertices.

   (a) Is D_n regular?

   (b) Explain why we know that $A + I = J$.

   (c) Hence or otherwise, calculate the eigenvalues of D_n and their multiplicities.

3. (a) State the Cayley–Hamilton Theorem.

   (a) State Cauchy-Schwarz Inequality.

   (b) State Pythagoras' Theorem.

4. Prove that for any vectors u and v the following is true:
   $u + v$ and $u - v$ are orthogonal iff $||u|| = ||v||$.

5. Find an orthonormal basis of $\mathbf{R}^3$ containing $(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3})$ using the standard dot product.

6. Consider an inner product space. Given a finite set X of vectors $x_1, \ldots, x_n$, define a matrix $M = (m_{ij})$ where $m_{ij} = \langle x_i, x_j \rangle$.

   (a) Complete the following: X is orthonormal if and only if M is ...

   (b) Complete the following: X is linearly independent if and only if M is ....