MthSc 853 — Goddard — Fall09

Practice Test 1

1. Give three disjoint bases for $\mathbf{R}^2$.

2. In a paragraph or two, outline the proof that for a finite-dimensional vector space V and operator $T: V \to W$ it holds that $\dim V = \dim \operatorname{null} T + \dim \operatorname{range} T$.

3. True or false:
   (a) For any subspace U it holds that $U + U = U$.
   (b) $\mathbf{C}^4$ is isomorphic to $\mathbf{R}^4$.
   (c) For any vector space V: If $S, T \in \mathcal{L}(V)$, and S, T both invertible, then ST invertible.
   (d) For any vector space V: If $S, T \in \mathcal{L}(V)$, and ST invertible, then S and T both invertible.

4. Give an example of spaces V and W such that $\dim \mathcal{L}(V, W) = 5$.

5. Consider the vector space $\mathcal{P}_2[\mathbf{R}]$. Let D be the operator differentiation (for example $D(3x^2 + 5) = 6x$).
   (a) Give a basis for $\operatorname{null} D$ and $\operatorname{range} D$.
   (a) Determine the matrix of D with respect to the standard basis $(1, x, x^2)$.
   (b) Explain what that tells us about the eigenvalues of D.
   (c) Determine the matrix of D with respect to the basis $(1 + x^2,\ 1 + x,\ x + x^2)$.

6. (a) Give an example of a space and operator where every nonzero vector is an eigenvector.
   (b) Give an example of a space and operator where no nonzero vector is an eigenvector.