MthSc 853 — Goddard — Fall09

Solution Sketch for Practice Test 2

1. $2(a+b+c)$ and $4abc$

2. (a) Yes

   (b) A has 0 on the diagonal and 1 everywhere else; I is the other way around. So their sum has 1s everywhere.

   (c) $n-1$ once and -1 $n-1$ times.

4. (a) If $u+v$ and $u-v$ are orthogonal, then by Pythagoras, $||u+v||^2 + ||u-v||^2 = ||2u||^2$ and $||u+v||^2+||v-u||^2 = ||2v||^2$. Since the left-hand-sides are equal, the right-hand-sides are equal. Thus $||u|| = ||v||$.

   (b) If $||u|| = ||v||$, then $\langle u+v, u-v\rangle = ||u||^2 + \langle v,u\rangle - \langle u,v\rangle - ||v||^2 = \langle v,u\rangle - \langle u,v\rangle = 0$, where the latter uses the fact we are in a real space.

   (c) Consider the complex vector space $\mathbf{C}$ with the standard dot product. Let $u = 1+2i$ and $v = 2+i$. Then $||u|| = ||v||$, but $\langle u+v, u-v\rangle = (3+3i)\overline{(i-1)} = -6i$.

5. Multiple answers including $(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3})$, $\frac{1}{\sqrt{2}}(0,1,1)$ and $(-\frac{2}{9}, -\frac{1}{18}, \frac{1}{18})$.

6. (a) the identity

   (b) nonsingular/invertible