Summary of Axler Chapter 7: Part I

We are working with operators over **C** or **R**. An operator is self-adjoint if $T = T^*$. If T is self-adjoint, then the eigenvalues are real.

Assume V is complex. Then operator T is self-adjoint iff $\langle Tv, v\rangle$ is real for all v.

Assume $\langle Tv, v\rangle = 0$ for all v. If V is complex or if T is self-adjoint, then $T = 0$.

An operator is normal iff $TT^* = T^*T$. An operator is normal iff $||Tv|| = ||T^*v||$ for all v. If $T \in \mathcal{L}(V)$ is normal, then (a) λ an eigenvalue for T iff $\bar{\lambda}$ an eigenvalue for T^*, and (b) eigenvectors of T from different eigenvalues are orthogonal.

Spectral Theorem: Over a complex space, V has an orthonormal basis of eigenvectors of T if and only if T is normal. Over a real space, V has an orthonormal basis of eigenvectors of T if and only if T is self-adjoint.