

Summary of Axler Chapter 3 (part 2)

A linear map can be represented as a matrix. If $T : \mathbf{F}^n \rightarrow \mathbf{F}^m$, then the matrix \mathcal{M} has m rows and n columns. Given basis $(v_j)_{j=1}^n$ for V and $(w_i)_{i=1}^m$ for W , the k^{th} column explains how to write Tv_k as a linear combination of (w_i) .

The addition of linear maps corresponds to the addition of the corresponding matrices. Scalar multiplication of linear maps corresponds to scalar multiplication of their matrices. The product of linear maps corresponds to the product of their matrices.

A linear map S is invertible iff there is a map T such that $ST = I$, where I is the identity map. This occurs if and only if the map is injective and surjective.

Any two vector spaces of dimension n are isomorphic. The proof idea is to start by mapping a basis to a basis and extending this to the whole space.

Given basis $(v_j)_{j=1}^n$ for V and $(w_i)_{i=1}^m$ for W , if \mathcal{M} is the matrix for any linear map, then \mathcal{M} is an invertible linear map between $\mathcal{L}(V, W)$ and $\text{mat}(m, n, \mathbf{F})$, the set of all $m \times n$ matrices with elements in \mathbf{F} . It follows that the dimension of $\mathcal{L}(V, W)$ is $(\dim V)(\dim W)$.

A linear map from V to itself is called an operator. If V is finite-dimensional, a map $T \in \mathcal{L}(V)$ is invertible if it is either injective or surjective.