GIRTH AND THE GREEDY ALGORITHM

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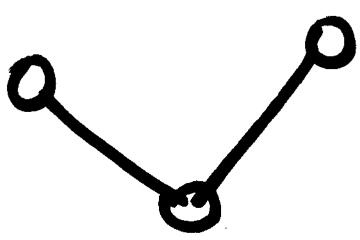
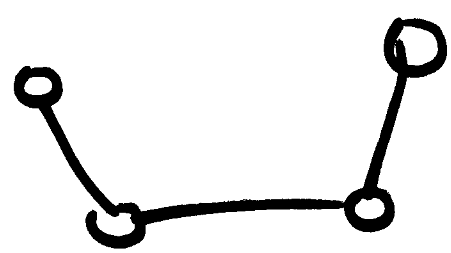
– when does the exercise of belt tightening become difficult?

1. Every maximal indept. set of vertices is of same cardinality (∴ maximum).

[well-covered : M. Plummer]

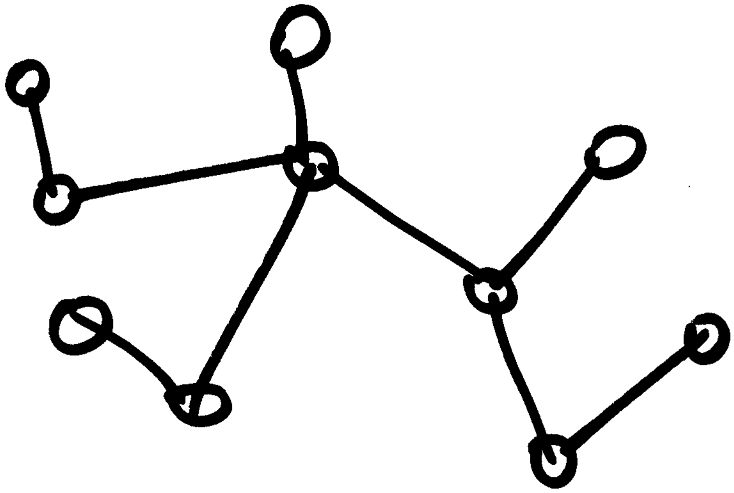


NO.
{1, 2}



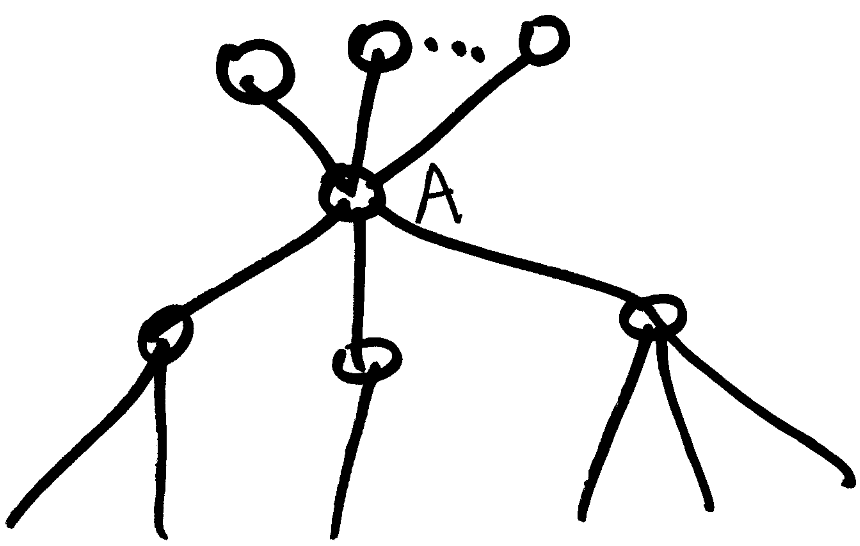
YES.
{2}

CONSIDER TREES . . .

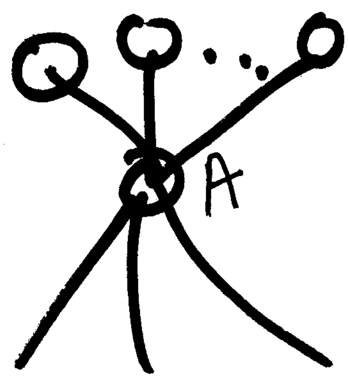


exactly 1 per subset

What about a node with several leaves?



Cannot have

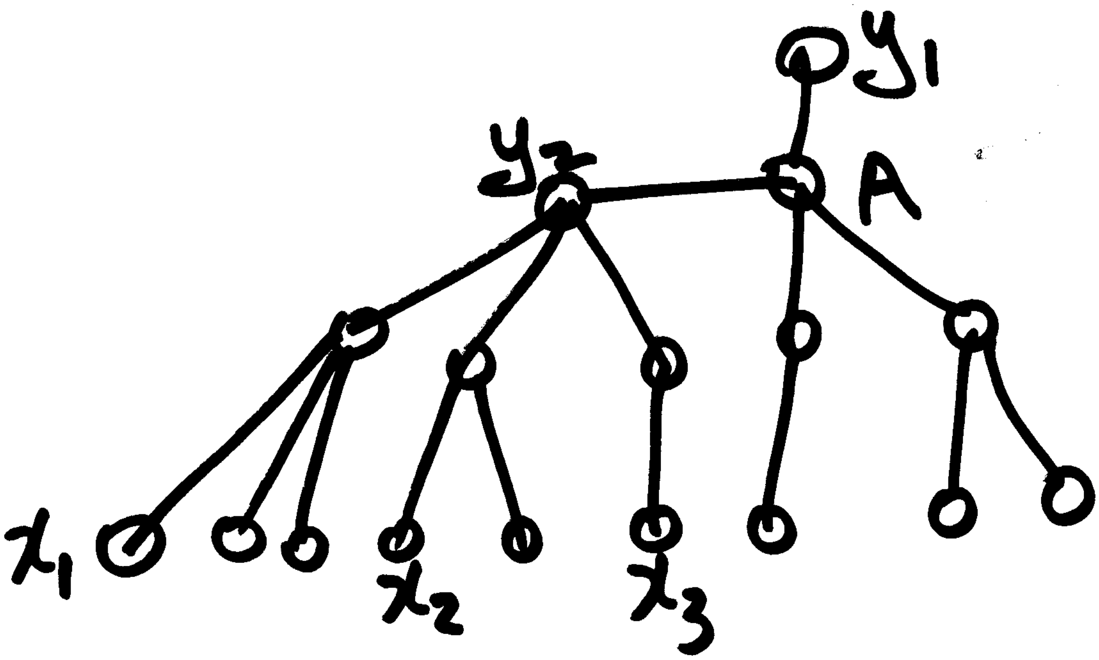


in any well-covered graph

(regardless of girth)

Extend A to a max'l indept. set. Then swap A for leaves attached to A.

How about some nodes with no leaves & some with leaves?

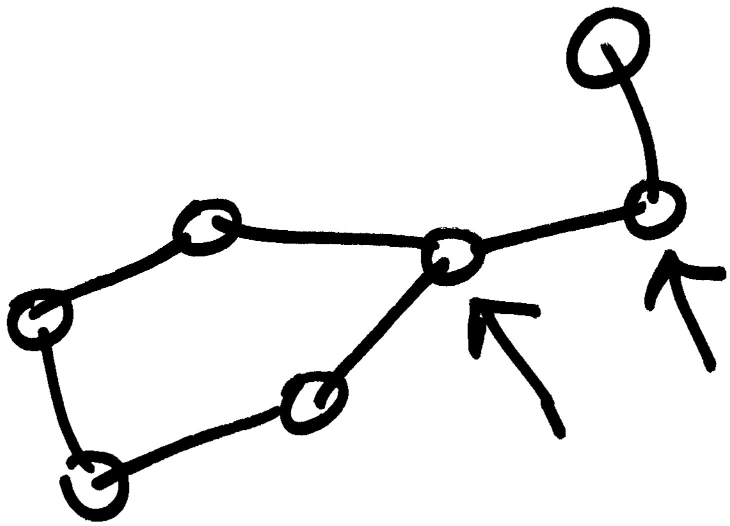


Extend $A \cup \{x_1, x_2, x_3\}$ to a maximal indept. set S.

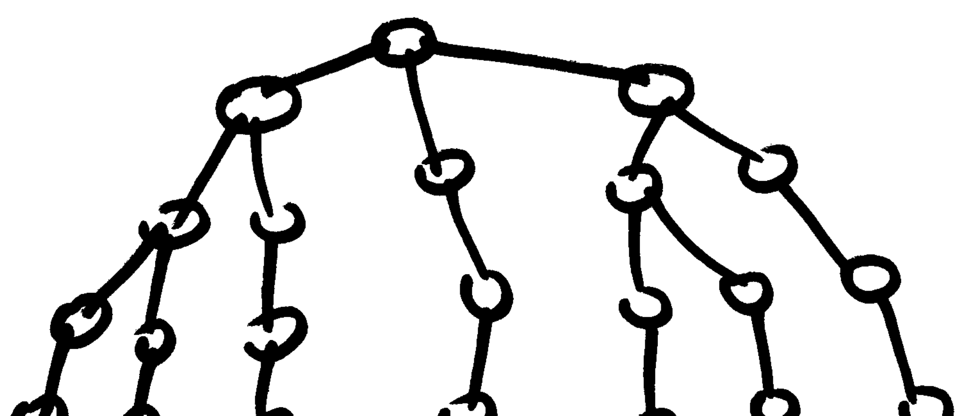
But $S \setminus \{A\} \cup \{y_1, y_2\}$ a larger indept. set.

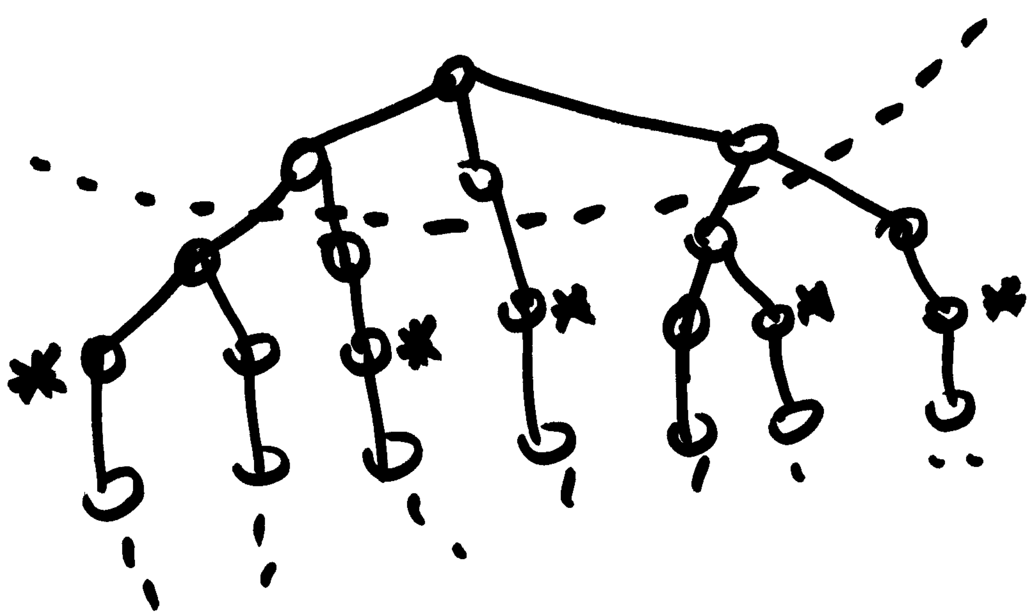
[works for girth ≥ 6]

NOTE: IF girth=5, not valid.

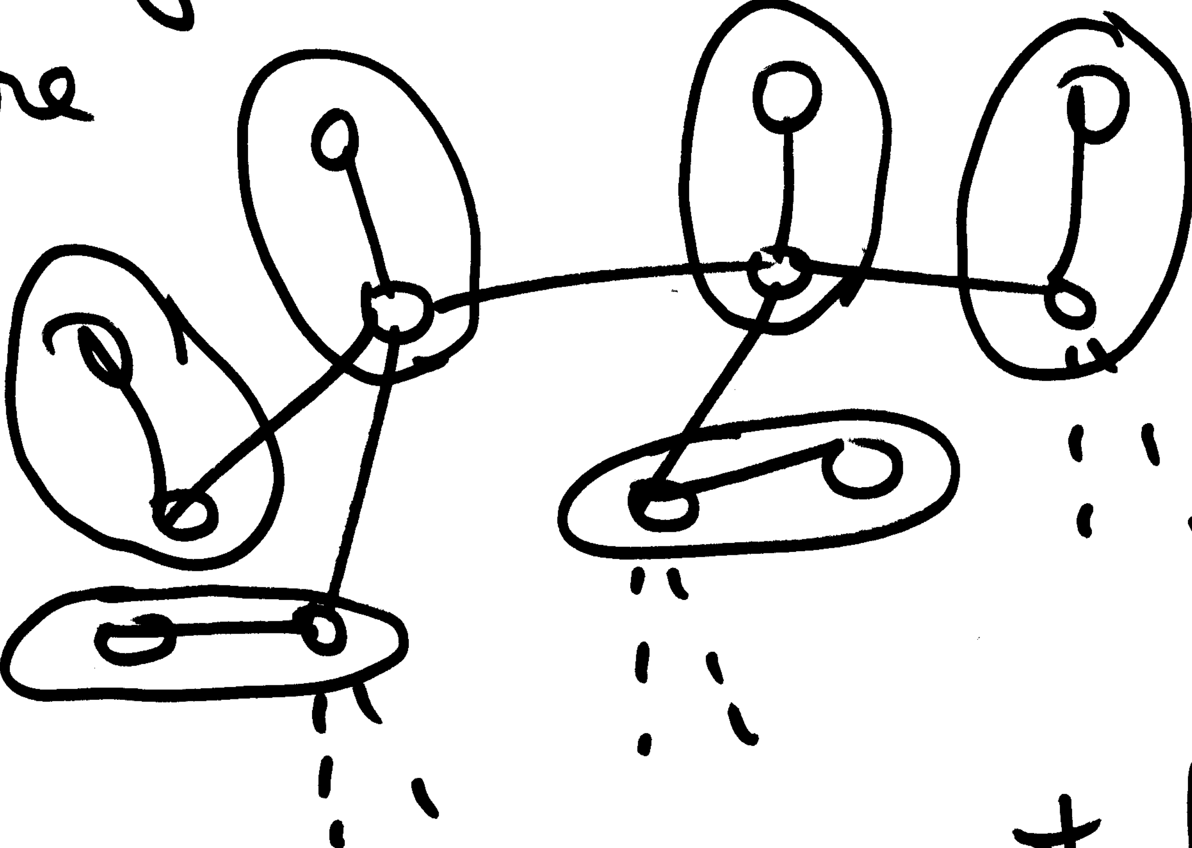


WHAT ABOUT NON-TREES? SAY MIN DEGREE ≥ 2.



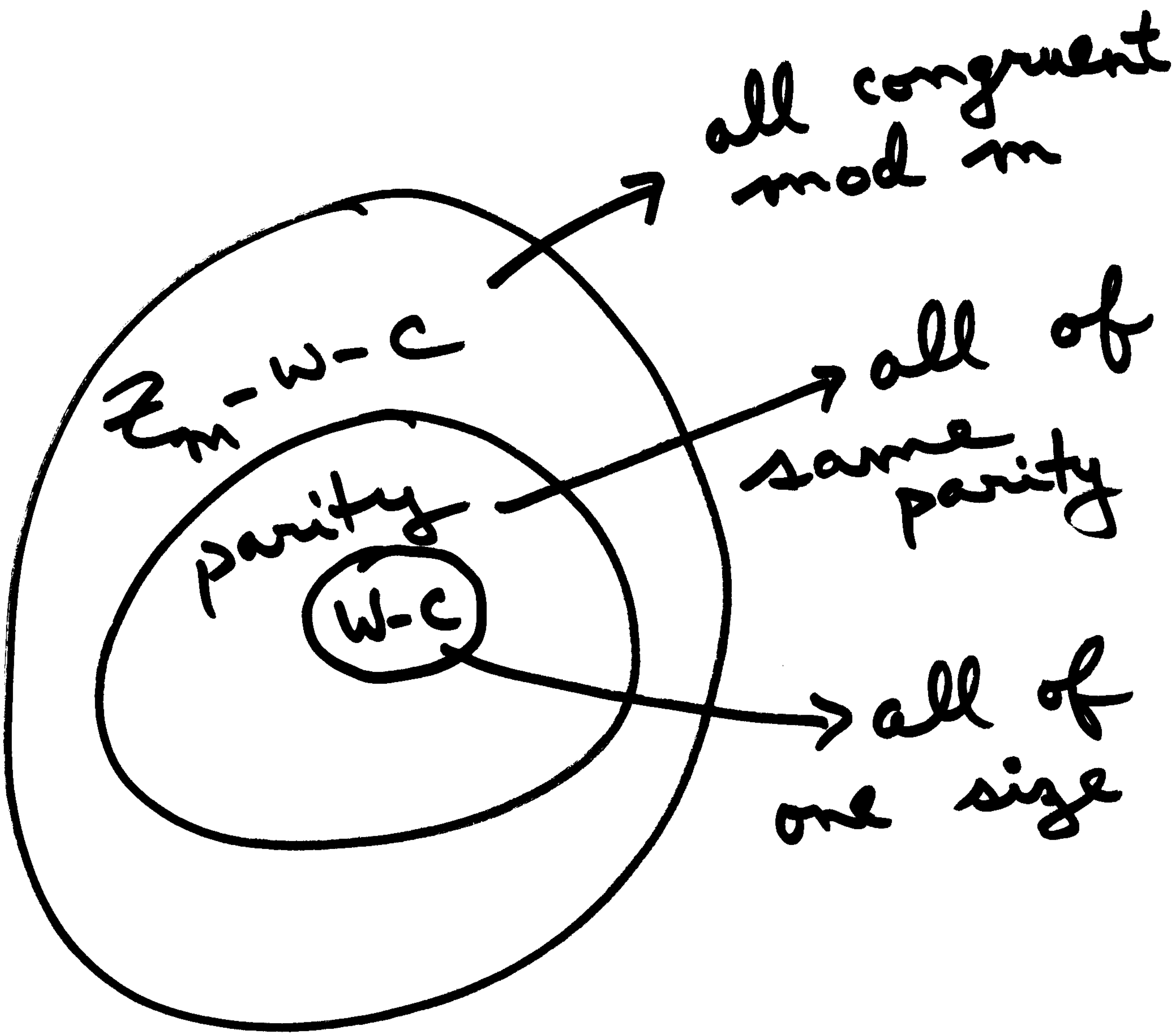


IF girth ≥ 8, the only well-covered graphs are



$+ K_1$

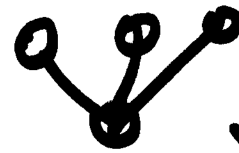
Sharp as C_7 well-covered.

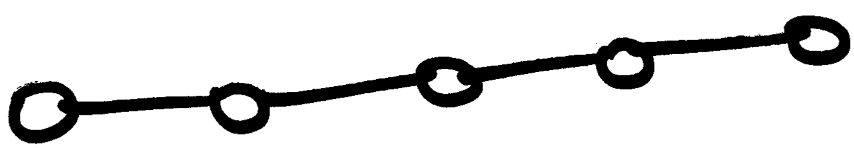


Tree char'n essentially "tells the story" for girth ≥ 8

M_t: Graphs that have exactly t different sizes of maximal indept. sets

M_1: well-covered

M_2: e.g.,  ,



M_2 graphs char'd for girth ≥ 8.

Tree char'n holds for girth ≥ 14.

(Sharp as $C_{13} \in M_2$)

independent set $\cong$ 1-packing
(all nodes ≥ 2 apart)

$\vdots$

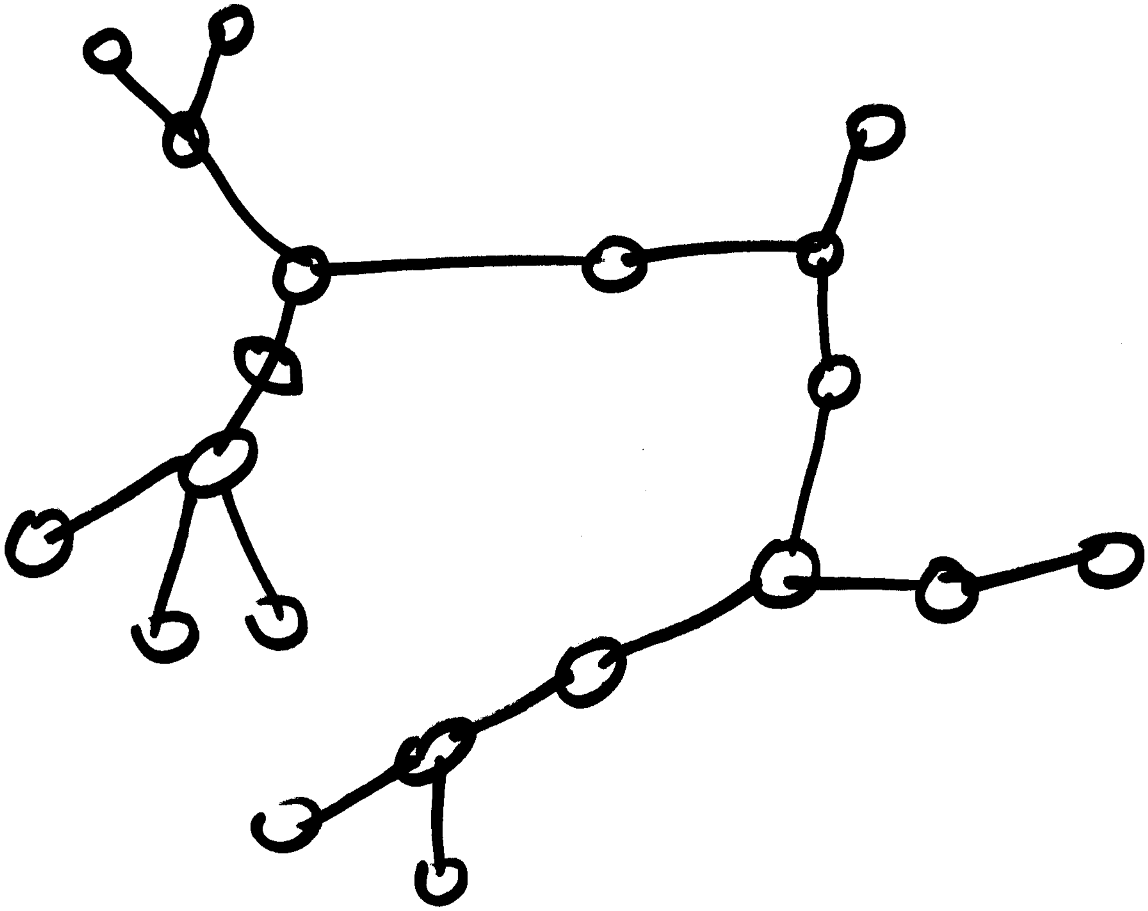
K-packing

(all nodes $\geq K+1$ apart)

The characterization of trees in which every maximal K-packing is maximum holds for all graphs of girth $\geq 4K+4$.

e.g., $K=1$ (well-covered) girth ≥ 8

$K=2$ girth ≥ 12



C_{11} also works so girth ≥ 12 is sharp.

K=1 : For girth ≥ 6 (only C_7 besides tree-like family)

K=2 : <u>suspect</u> for girth ≥ 9, only C_{11} besides tree-like group ?

2. Consider packing copies of H into a graph G (each edge can be used only once).

— Much work on decomposing G into copies of H (∴ using each edge exactly once).

e.g., $K_n \to K_t$'s etc.

— Also work on characterizing graphs with property that one can delete the edges of a number of edge disjoint copies of the subgraph, &, regardless of how done, the graph remaining can still be decomposed.

randomly packable or

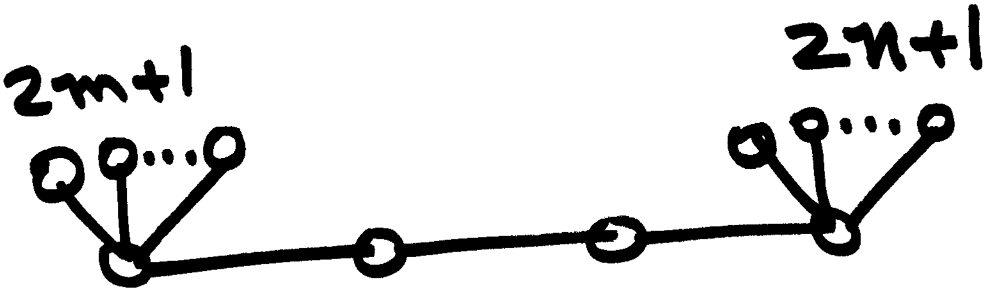
randomly decomposable

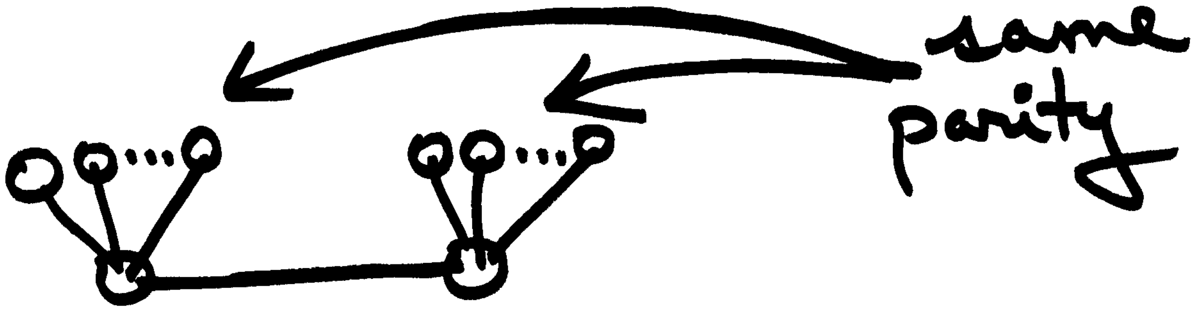
Now consider, given a graph H, which graphs (call them _equipackable_) have property that every maximal edge disjoint packing with H is maximum.

$H \cong K_2$ ☺

$H \cong P_3$?

e.g.,





For girth ≥ 8, "that's all folks".

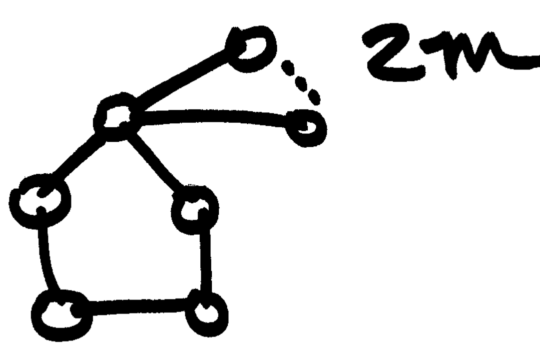
$\vdots$

C_7 also in collection.

nothing with girth $= 6$.

girth ≥ 5, C_5

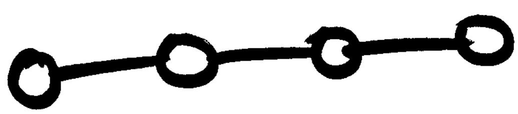
+

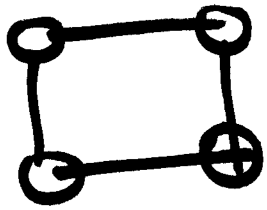


3. CONSIDER 2-PERSON GAME: TAKE TURNS REMOVING AN EDGE FROM G. CANNOT ISOLATE A VERTEX. LAST PLAYER ABLE TO MOVE WINS.

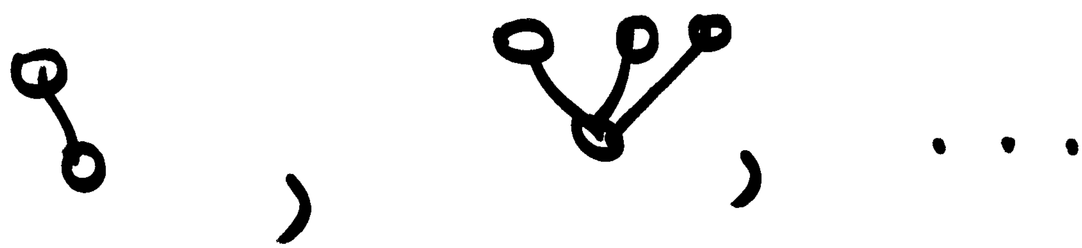
⋮

WHAT ABOUT GRAPHS THAT REGARDLESS OF HOW MOVES MADE, THE SAME # OF EDGES DELETED?

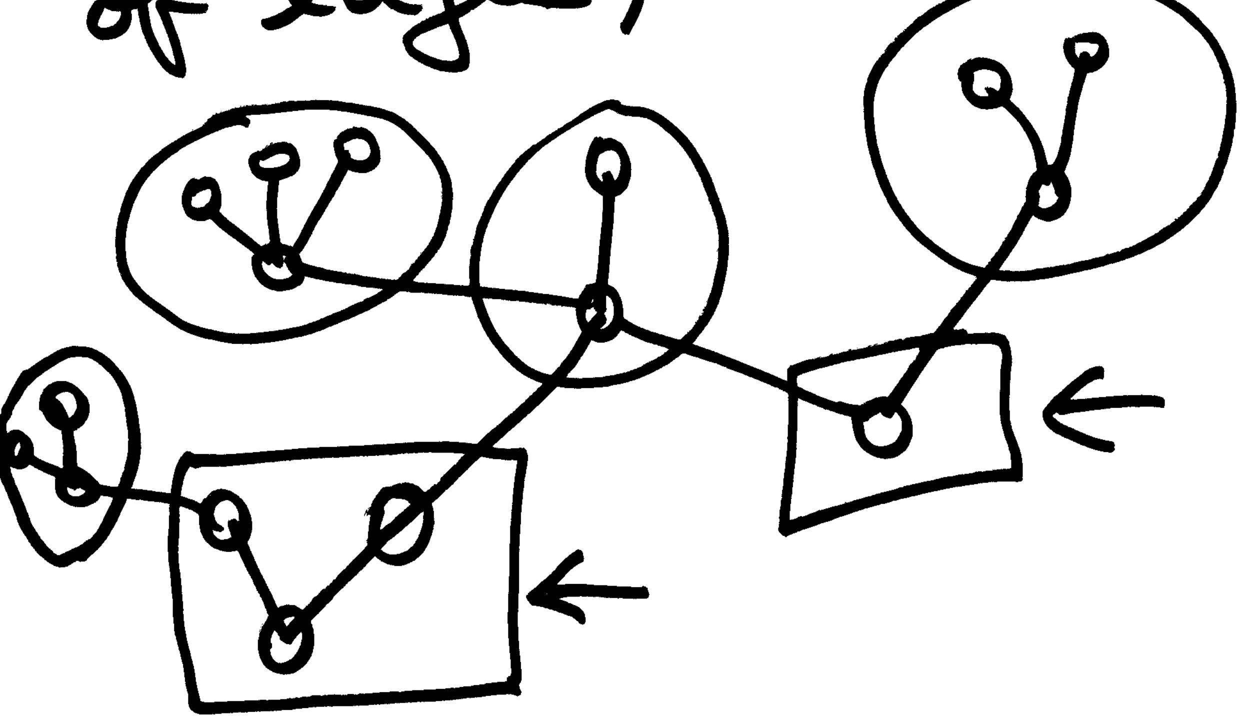
e.g.,  → 1

 → 2

NOTE: FINISHED WHEN FACING A COLLECTION OF STARS



(note: want those graphs with property that every star factor has same # of edges)

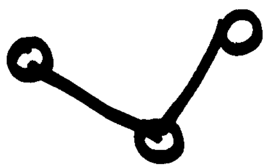


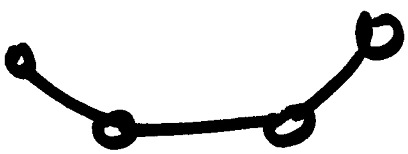
tree char'n works for graphs of girth ≥ 8.

C_7 again!

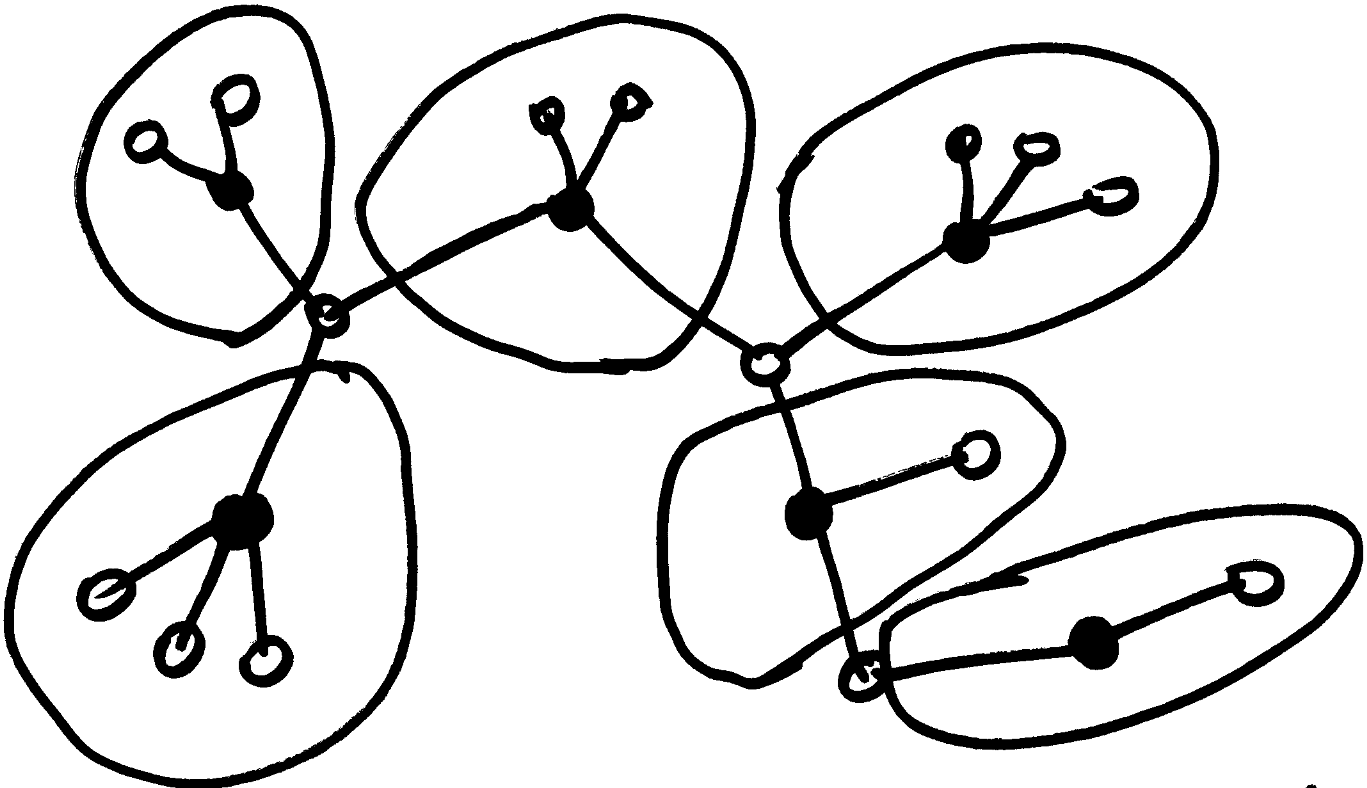
girth ≥ 6, C_7 only exception to tree-like family.

4. Consider graphs with the property that every minimal edge-dom. set (edges dominating edges) is minimum.

 ← YES $\{1\}$

 ← NO $\{1,2\}$

Let S_i be set of edges incident with stem/support node i. If these sets form a partition of $E(G)$, then G has required property.



must have exactly 1 from each blue set.

For any girth this condition is suff. (to guarantee in family).

For girth ≥ 8, necessary.

C_7 shows sharp.