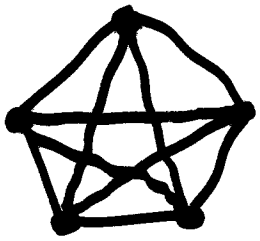
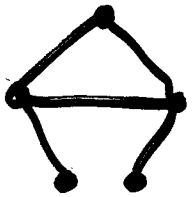
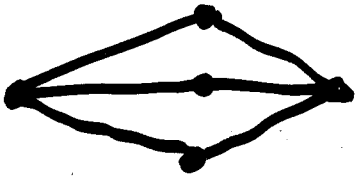
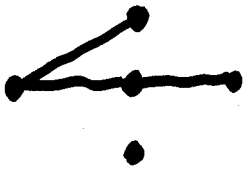
Forced Relatively Self Complementary Graphs

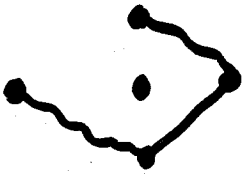
Dean Hoffman, Auburn U.



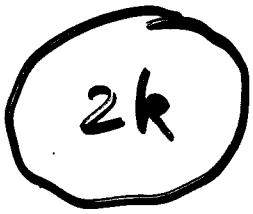


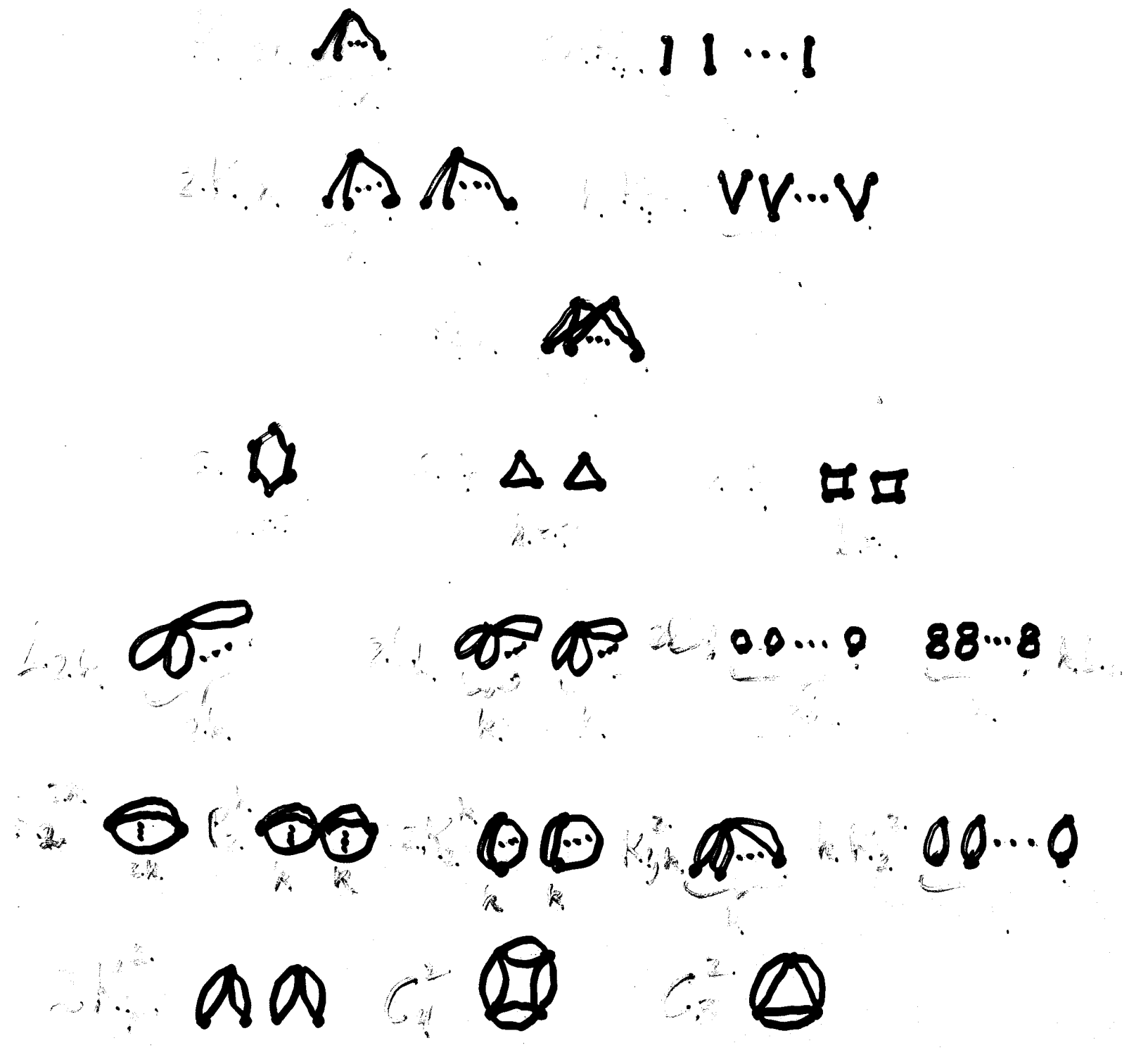






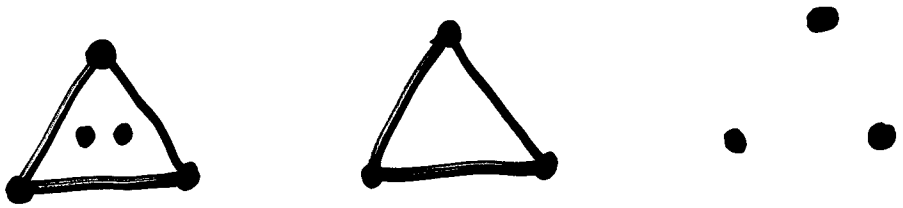
If k is a positive integer, the graph G is said to be *k-good* if it has 2k edges, any partition of the edge set into two parts of size k produce isomorphic subgraphs.





We conjecture these are the only k-good graphs!

In what follows, we will ignore isolated vertices. For example:



G_1 G_2 G_3

G_1 is connected, and isomorphic to G_2. And G_3 is not a graph, as it has no vertices!

We define the vector a of positive integers to be *k-good* if $\sigma(a) = 2k$, and for all non-negative integer vectors b, c with $\sigma(b) = \sigma(c) = k$ and $b + c = a$, b and c can be obtained from each other by permuting coordinates. Some examples:

$(2k)$, (k, k), $(1,1, \ldots, 1)$, $(2, 2, \ldots, 2)$

$2k$ k

Theorem 1 Those are the only k-good vectors.

Theorem 2 Let C_i be the components of a k-good graph, and let a be the vector with coordinates $|E(C_i)|$. Then a is a k-good vector.

Hint: If H and G are connected graphs with $H \leq G$, and if $|E(H)| \leq x \leq |E(G)|$, then there is a connected graph K, with $|E(K)| = x$, and $H \leq K \leq G$.

Theorem 3 The components of a k-good graph are isomorphic to each other.

Theorem 4 If a k-good graph has two components, it's on the list.

Theorem 5 A k-good graph cannot contain both loops and links.

Theorem 6 If G is a k-good loopy graph, it corresponds to a k-good vector, and hence is on the list.

Theorem 7 If G is a loopless k-good graph, form the vector a as follows: for each pair u, v of adjacent vertices, the number of edges joining u and v is a coordinate of a. Then a is a k-good vector.

Theorem 8 If G is a non-simple k-good graph, it's on the list.

Theorem 9 If G is a k-good graph with a vertex of degree at least k, it's on the list.

Theorem 10 If G is a k-good graph with a cut vertex, it's on the list.

Final Summary:

If G is a k-good graph not on the list, then G is simple, two-connected, and all it's vertices have degree strictly less than k.

That's all folks!